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Abstract

An autopilot can be used to provide precise control to meet the demanding requirements of flight research maneuvers with high-performance aircraft. This paper presents the development of control laws within the context of flight test maneuver requirements. The control laws are developed using eigensystem assignment and command generator tracking. The eigenvalues and eigenvectors are chosen to provide the necessary handling qualities, while the command generator tracking enables the tracking of a specified state during the maneuver. The effectiveness of the control laws is illustrated by their application to an F-15 aircraft to ensure acceptable aircraft performance during a maneuver.

Introduction

Conventional piloting techniques are often inadequate to meet the demanding requirements of flight research maneuvers with high-performance aircraft. These maneuvers often require precise control of onset rates in extreme flight conditions. Thus the pilot may be trying to control an aircraft at high angles of attack and high g's while attempting to increase normal acceleration at a prescribed rate through a maneuver specified at the very limits of accuracy of the cockpit instruments.

A new flight test technique[1] was developed at the Dryden Flight Research Facility of the NASA Ames Research Center (Ames-Dryden) to aid the pilot during these maneuvers. The essence of this technique is the application of an autopilot to provide precise control during the required flight test maneuvers. The flight test maneuver autopilot (FTMAP) is designed to provide precise, repeatable control of a high-performance aircraft during certain prescribed maneuvers so that a large quantity of data can be obtained in a minimum of flight time.

The FTMAP can be used for various maneuvers, including straight-and-level flight, level accelerations and decelerations, pushover pullups, excess-thrust windup turns, thrust-limited turns, and the "rockinghorse" maneuvers. Each of these maneuvers involves tracking certain states of the aircraft and holding certain states within prescribed values, as well as placing constraints on the derivatives of the states (for example, the excess-thrust windup turn is performed at constant altitude and Mach number with angle of attack increasing at a specified rate to achieve the final angle of attack).

This paper discusses the development of the FTMAP control laws within the context of flight test maneuver requirements. The FTMAP control laws are developed using the eigensystem assignment[2] and command generator tracking.[3] The feedback gain obtained from eigensystem assignment ensures the desirable performance of the aircraft. The feedforward methodology of command generator tracking ensures the tracking of desired states during a particular maneuver. The desired closed-loop eigenvalues are taken from MIL-F-8785-C,[4] and each of these eigenvalues is assigned an eigenvector that distributes the modal response among the state variables and outputs of the system. However, each eigenvector is constrained to lie in an m-dimensional subspace[5] (m is the number of independent controls), and an element in this subspace is selected by finding the best linear projection of an unconstrained desired vector in this subspace. A feedback gain matrix, K, is computed using the eigenvalues and achievable eigenvectors.

The feedforward gains are computed by using the command generator tracking of Broussard.[3] This method would ensure the tracking of a state trajectory while the aircraft is performing a particular maneuver. The tracking objective can be described in terms of the controlled variables of the aircraft being able to follow the output of a model. The input to the model, which is the pilot input, is assumed to be constant and the output of the model is the desired state trajectory the aircraft will follow while undergoing a maneuver.

The effectiveness of the developed control laws is illustrated in this paper by application to an F-15 aircraft.

Eigensystem Synthesis

Two widely used synthesis techniques of modern control theory are the linear quadratic regulator design and the modal control theory involving pole placement or eigenvalue/eigenvector assignment. One of the purposes of feedback control of aircraft is to improve or enhance the flying qualities of an aircraft. The difficulty in incorporating specifications such as damping, natural frequency, and decoupling within a quadratic performance index makes the eigensystem synthesis procedure a

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promising design alternative. The performance specifications can be interpreted in terms of desired closed-loop eigenvalues and eigenvectors. Moore[6] and others have shown how feedback can be used to place closed-loop eigenvalue and shape closed-loop eigenvectors. Cunningham,[7] Andry, Shapiro, and Chung,[8] and Sobel and Shapiro[9] have successfully demonstrated the use of eigenstructure assignment procedure for aircraft control system design.

The handling qualities data base may be used to obtain desired pole locations directly. The additional design objective of obtaining augmented dynamics similar to those obtained in flight leads to specifications on eigenvectors or desired mode shapes. For example, pitch attitude must be dominant for the short-period mode, and speed must be dominant for the phugoid mode.

Detailed discussions on eigenspace can be found in Ref. 8, but some basic results for controllable and observable systems are summarized in the following discussion.[10]

Consider the system

$$\dot{x} = Ax + Bu \qquad (1)$$

$$y = Cx \qquad (2)$$

where $x \in R^n$, $u \in R^m$, and $y \in R^r$. A, B, and C are matrices of appropriate dimensions. If the system is controllable and observable, and the matrices B and C are of full rank, the following results hold.

1. The positions of maximum (m, r) closed-loop eigenvalues can be assigned arbitrarily with the stipulation that if λ_i is a complex closed-loop eigenvalue, its complex conjugate, λ_i^*, must also be a closed-loop eigenvalue.

2. The shape of maximum (m, r) eigenvectors can be altered. If the shape of a complex eigenvector v_i is altered, its complex conjugate, v_i^*, must be altered in the same way.

3. For each eigenvector whose shape is altered, minimum (m, r) eigenvector elements can be chosen arbitrarily.

4. Attainable eigenvectors must lie in the subspace spanned by the columns of $(\lambda_i I - A)^{-1}B$ of dimension m, which is the number of independent control variables. A desired eigenvector, v_i^d, will, in general, not reside in the prescribed subspace and cannot be achieved. The achievable eigenvector, v_i^a, is obtained by orthogonal projection of v_i^d onto the subspace spanned by $(\lambda_i I - A)^{-1}B$. It will generally be true that

only a few of the components in v_i are actually specified. The rest can be arbitrary. To account for this, we reorder and partition v_i as follows:

$$[v_i]^{R_i} = \left(\frac{v_i^*}{d_i} \right) \qquad (3)$$

where

v_i^* is the specified subvector

d_i is the vector of unspecified components and

$[. . .]^{R_i}$ is the reordering operation

If we let

$$\left(\frac{v_i^o}{d_i} \right) = [(\lambda_i I - A)^{-1}B]^{R_i} Z_i = \left(\frac{L}{M} \right) Z_i \qquad (4)$$

then, as shown in Ref. 5, we can select Z_i to best approximate v_i^* with v_i^o and by method of orthogonal projections obtain

$$Z_i = (L'L)^{-1} L'v_i^* \qquad (5)$$

As shown by Moore,[6] the feedback gain K is given by

$$K = [w_1 \ w_2 \ . \ . \ . \ w_n][v_1 \ v_2 \ . \ . \ . \ v_n]^{-1} \qquad (6)$$

where w_i is obtained from the relation

$$[\lambda_i I - A]v_i = Bw_i \qquad (7)$$

Command Generator Tracking

The development of control laws based on command generator tracking is discussed by Broussard.[3] Briefly, if a model specifying the desired behavior of an aircraft is defined by

$$\dot{x}_m = A_m x_m + B_m u_m \qquad (8)$$

$$y_m = C_m x_m + D_m u_m \qquad (9)$$

and the variables of the aircraft to be tracked are described by

$$y_t = Hx \qquad (10)$$

a control law must be found such that $y_t \approx y_m$. If x^* and u^* are the ideal aircraft state and input, respectively, that allow perfect tracking, then

$$x^* = S_{11}x_m + S_{12}u_m \qquad (11)$$

$$u^* = S_{21}x_m + S_{22}u_m \qquad (12)$$

The results in Eqs. (11) and (12) assume that the input to the model, u_m, is a step input. However, it does not mean that only a constant command input is being tracked. Any command signal that can be described as a solution of the differential equation forced by a step input (or zero) can be used, provided it is augmented to the model state and not to the model input.[11] The matrices S_{11}, S_{12}, S_{21}, and S_{22} are given by

$$S_{11} = \Omega_{11}S_{11}A_m + \Omega_{12}C_m$$

$$S_{12} = \Omega_{11}S_{11}B_m + \Omega_{12}D_m$$

$$S_{21} = \Omega_{21}S_{11}A_m + \Omega_{22}C_m$$

$$S_{22} = \Omega_{21}S_{11}B_m + \Omega_{22}D_m \qquad (13)$$

and

$$\Omega = \begin{bmatrix} \Omega_{11} & | & \Omega_{12} \\ \hline \Omega_{21} & | & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A & | & B \\ \hline H & | & 0 \end{bmatrix}^{-1} \qquad (14)$$

To incorporate the state feedback into the design, if we let

$$\tilde{x} = x - x^* \qquad (15)$$

$$\tilde{u} = u - u^* \qquad (16)$$

then

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u} \qquad (17)$$

and

$$\tilde{u} = K(x - x^*) \qquad (18)$$

or

$$u = u^* + \tilde{u} = S_{21}x_m + S_{22}u_m + K(x - x^*)$$

$$= S_{21}x_m + S_{22}u_m + Kx$$

$$- K(S_{11}x_m + S_{12}u_m) \qquad (19)$$

Equation (19) gives the control to be applied to the aircraft, depending on the maneuver of the aircraft. For example, for constant-level acceleration, the command velocity is to increase linearly while the command altitude is held constant. The input to the model is constant, while the outputs of velocity and altitude are a ramp function and zero, respectively. In this case,

$$A_m = 0$$

$$B_m = I$$

$$C_m = I$$

$$D_m = 0 \qquad (20)$$

with an appropriate value for u_m. In addition,

$$S_{11} = \Omega_{12}$$

$$S_{12} = \Omega_{11}\Omega_{12}$$

$$S_{21} = \Omega_{22}$$

$$S_{22} = \Omega_{21}\Omega_{12} \qquad (21)$$

$$x^* = \Omega_{12}x_m + \Omega_{11}\Omega_{12}u_m \qquad (22)$$

$$u^* = \Omega_{22}x_m + \Omega_{21}\Omega_{12}u_m \qquad (23)$$

and

$$u = \underbrace{(\Omega_{22} - K\Omega_{12})x_m + (\Omega_{21}\Omega_{12} - K\Omega_{11}\Omega_{12})u_m}_{\text{Feedforward gain}}$$

$$+ Kx \qquad (24)$$

Example

The feedback and feedforward control technique developed by eigensystem and command generator tracking synthesis technique is applied for the control of an F-15 aircraft. Ames-Dryden's detailed nonlinear aerodynamic model of the F-15, linearized by trimming the aircraft at the desired flight condition and deriving linear models by numerical perturbation, is used. The linear model is represented in the state equation form as

$$\dot{x} = Ax + Bu$$

where

$$x = \begin{bmatrix} v \\ \alpha \\ q \\ \theta \\ h \\ \beta \\ p \\ r \\ \phi \end{bmatrix} \begin{array}{l} \text{velocity} \\ \text{angle of attack} \\ \text{pitch rate} \\ \text{pitch angle} \\ \text{altitude} \\ \text{angle of sideslip} \\ \text{roll rate} \\ \text{yaw rate} \\ \text{roll angle} \end{array}$$

$$u = \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \\ \delta_t \end{bmatrix} \begin{array}{l} \text{aileron deflection} \\ \text{elevator deflection} \\ \text{rudder deflection} \\ \text{throttle} \end{array}$$

For the flight condition corresponding to an altitude of $h = 20,000$ ft and a Mach number (M) of 0.8, the values for A and B matrices of Eq. (1) are as follows.

$$
A = \begin{bmatrix}
-0.0108 & 24.1966 & 0.0000 & -32.1129 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
-0.0001 & -1.0942 & 1.0000 & 0.0001 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
-0.0001 & -3.2862 & -2.1922 & 0.0009 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & -829.5390 & 0.0000 & 829.5390 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.2337 & 0.0358 & -0.9994 & 0.0387 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -40.0103 & -2.1420 & 1.2406 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 9.0098 & -0.0340 & -0.6040 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0358 & 0.0000
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0.0000 & -1.0734 & 0.0000 & 0.3792 \\
0.0000 & -0.1504 & 0.0000 & 0.0000 \\
0.0000 & -16.1223 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 \\
-0.0022 & 0.0000 & -0.0388 & 0.0000 \\
13.5934 & 0.0000 & -1.4674 & 0.0000 \\
0.1488 & 0.0000 & -4.5577 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000
\end{bmatrix}
$$

The open-loop eigenvalues of the aircraft are

$\left.\begin{array}{l} -1.6433 + 1.7278i \\ -1.6433 - 1.7278i \end{array}\right\}$ Dutch roll mode

$\left.\begin{array}{l} -0.4088 + 3.2303i \\ -0.4088 - 3.2303i \end{array}\right\}$ Short period mode

$-2.1413 - 0.0000i$ Roll subsidence mode

$\left.\begin{array}{l} -0.0054 + 0.0394i \\ -0.0054 - 0.0394i \end{array}\right\}$ Phugoid mode

$0.0000 + 0.0000i$ Altitude mode

$-0.0209 + 0.0000i$ Spiral mode

The desired closed-loop eigenvalues are selected as follows

$\left.\begin{array}{l} -2.0000 + 4.0000i \\ -2.0000 - 4.0000i \end{array}\right\}$ Dutch roll mode

$\left.\begin{array}{l} -1.0000 + 3.0000i \\ -1.0000 - 3.0000i \end{array}\right\}$ Short period mode

$-0.5000 + 0.0000i$ Altitude mode

$\left.\begin{array}{l} -0.0500 + 0.5000i \\ -0.0500 - 0.0500i \end{array}\right\}$ Phugoid mode

$-4.0000 + 0.0000i$ Roll subsidence mode

$-0.0020 + 0.0000i$ Spiral mode

The desired eigenvectors based on desired decoupled aircraft response are specified as follows, in the same order as the preceding eigenvalues.[5]

$$
\begin{array}{c}
\begin{bmatrix} v \\ \alpha \\ q \\ \theta \\ h \\ \beta \\ p \\ r \\ \phi \end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 & X & 1 & X & 0 & 0 \\
0 & 0 & X & 1 & X & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & X & 0 & X & X & 0 & 0 \\
0 & 0 & X & X & 0 & X & X & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & X & 1 & 0 & 0 \\
1 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & X \\
X & 1 & 0 & 0 & 0 & 0 & 0 & 0 & X \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 1
\end{bmatrix} \\
\underbrace{\qquad}_{\substack{\text{Dutch} \\ \text{roll}}} \underbrace{\qquad}_{\substack{\text{Short} \\ \text{period}}} \underbrace{\qquad}_{\text{Phugoid}}
\end{array}
$$

(x-arbitrary)

The feedback gain, K, based on achievable eigenvectors and closed-loop eigenvalues, is given by

$$
K = \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \\ \delta_t \end{bmatrix}
\begin{array}{ccc}
v & \alpha & q \\
-0.0000 & 0.0000 & 0.0000 \\
0.0038 & 0.0192 & -0.0442 \\
0.0000 & 0.0000 & 0.0000 \\
-0.0204 & 4.1420 & -10.9580
\end{array}
$$

$$
\begin{array}{ccc}
\theta & h & \beta \\
0.0000 & 0.0000 & 2.7167 \\
0.3850 & 0.0002 & 0.0000 \\
0.0000 & 0.0000 & -2.0771 \\
-2.6343 & -0.0010 & 0.0000
\end{array}
$$

$$
\begin{array}{ccc}
p & r & \phi \\
-0.1422 & -0.0147 & -0.0046 \\
0.0000 & 0.0000 & 0.0000 \\
-0.0483 & 0.7094 & -0.0329 \\
0.0000 & 0.0000 & 0.0000
\end{array}
$$

Two sets of trajectories are shown giving the results of the application of developed control laws to the aircraft. In the first instance, it is necessary to maintain the aircraft at trim conditions of the specified flight conditions. No feedforward is required in this case. Figures 1 to 4 show the state and control trajectories of the aircraft. The lateral states and the aileron and rudder inputs show no change from their trim values.

In the second case, it is necessary to accelerate the aircraft from Mach 0.8 to Mach 0.9, with a Mach rate of 0.01 Mach/sec, while maintaining aircraft altitude at 20,000 ft. Once the desired velocity of Mach 0.9 is reached, it is held constant. Figure 5 shows the variation of velocity of the aircraft when feedforward is also used. It shows an excellent tracking of the velocity. Figure 6 shows the corresponding variation in the altitude of the aircraft.

Concluding Remarks

This paper has presented a synthesis technique that could be extended for control of an aircraft undergoing a specified maneuver. Eigenvalue/eigenvector assignment procedure provides the necessary decoupling in aircraft handling qualities, while ensuring the location of eigenvalues to meet the specifications. The command generator tracking ensures the tracking of the controlled variables of the aircraft, as dictated by the requirements of a particular maneuver. The problem of developing suitable models to match the state trajectories of a given maneuver is being investigated further. In some cases this would involve gain scheduling and update of aircraft parameters, while ensuring that the constraints on the control surfaces are not violated.

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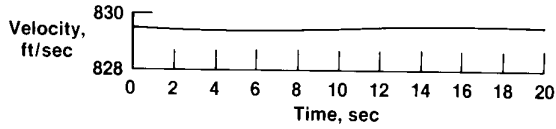
Fig. 1 Variation of velocity with time.
Mach = 0.8; desired altitude = 20,000 ft;
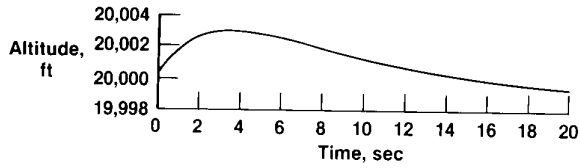aircraft level.

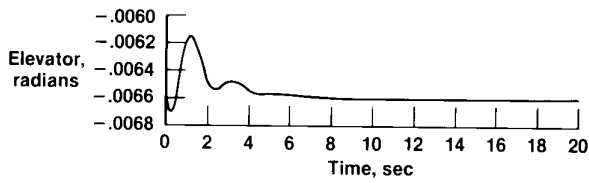Fig. 2 Variation of altitude with time.

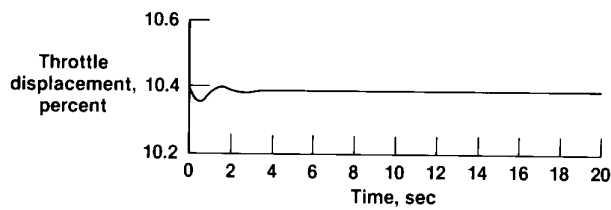Fig. 3 Elevator input to hold aircraft steady.

Fig. 4 Thrust input for holding aircraft steady.
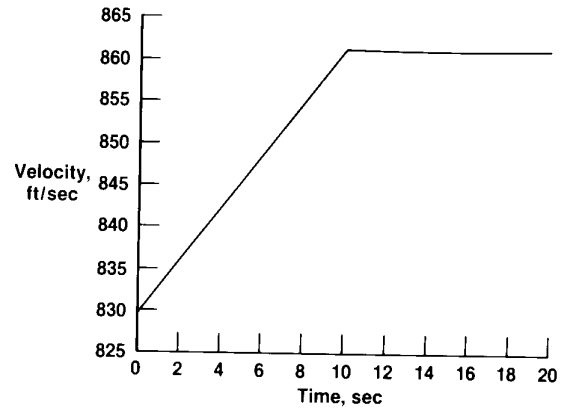

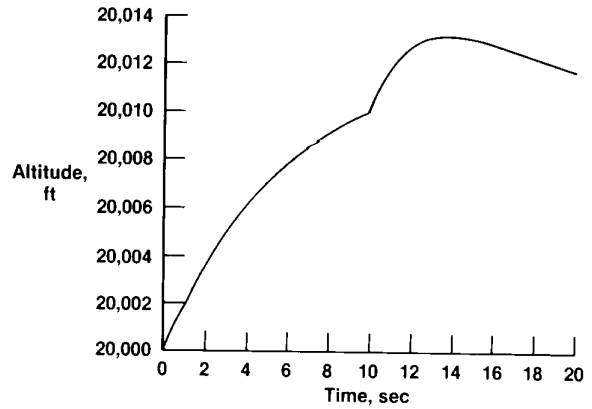Fig. 5 Variation of velocity for level acceleration from Mach 0.5 to Mach 0.8.

Fig. 6 Variation in altitude for level acceleration.

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